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Band Structures of a Photonic Crystal Waveguide with Koch Snowflake Fractal Structures

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Abstract. Many applications used today are based on the study of certain geometric tools; for example, a peculiar geometry known as fractals. In this work an integral method was developed to calculate the band structures of a photonic crystal waveguide, formed by two parallel conducting plates and an array of inclusions involving Koch snowflake fractal structures. The numerical technique is known as the Integral Equation Method, which starts from Green's second identity to solve the two-dimensional Helmholtz equation. We found that varying the inclusion size for several iterations of the Koch fractal structure allows us to control the band structure of the system. The results show the appearance of several band gaps that substantially modify the photonic band structure. Furthermore, it is possible to obtain discrete modes for a certain frequency range and then the periodic photonic crystal waveguide acts as an unimodal filter. These optical properties exhibit some interest from a technological point of view.

Keywords: Photonic band structures, Koch snowflake, band gaps, integral equation method.

1 Introducción

By analyzing certain geometric tools with the aim of being used in many research methods, this leads to the discovery of applications of great interest [2]. For example, a very peculiar geometry known as fractals, these appear both in nature and in the exact sciences [1]. Scattering of light by fractal surfaces has attracted some attention over the years, with the research reported so far being based on approximate approaches to the scattering equations or with reentrant fractals [4].

On the other hand, the determination of the band structure, reflectance, and transmittance of one- and two-dimensional photonic crystals with a complex unit cell structure, such as fractal geometries, has been developed based on the solution of integral equations [3]. In this context, we present a theoretical and numerical study of the electromagnetic response of a photonic crystal waveguide (PCW) based on the adoption Koch snowflake fractal structures.

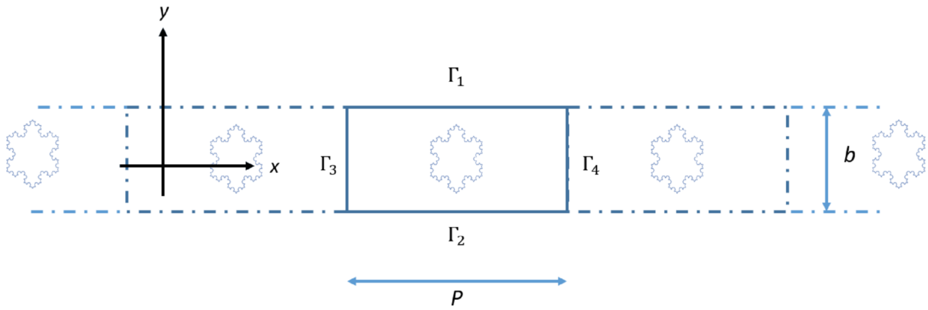

Fig. 1. Schematic description of a periodic waveguide with inclusions formed with perfectly-conductive Koch fractal structures.

To solve this problem, it was done using a numerical technique known as the Integral Equation Method (IEM) [3, 5], which starts from Green's second identity to solve the two-dimensional Helmholtz equation. This paper is organized as follows. In Sec. 2 we introduce an integral method for calculating the dispersion relation to determine the band structures of PCW with Koch snowflake fractal structures, based on ideas described in [3, 5]. Sec. 3 shows the numerial results of band structures of the considered system for different inclusion sizes with several iterations of the Koch fractal structures. Finally, Sec. 4 presents our conclusions.

2 Theoretical Approach

We consider a two-dimensional PCW, formed by two flat internal walls that enclose an array of Koch snowflake fractal structures. The surfaces involved are perfectly-conductive materials and the medium between the walls and the inclusions is vacuum. The geometry of the system is sketched in Fig. 1. In PCW we consider a period P in the flat profiles, a separation between the plates of the waveguide plates given by b and the Koch fractal inclusions for a given iteration, which can be in terms of the side length L of the original triangle.

2.1 Integral Equation Method

Assuming a time dependency $e^{-i\omega t}$ for electromagnetic fields, the wave equation can be transformed into the Helmholtz equation:

$$\nabla^2 \Psi_j(\mathbf{r}) + n_j^2(\omega)\frac{\omega^2}{c^2}\Psi_j(\mathbf{r}) = 0, \tag{1}$$

where j indicates the j-th medium with refractive index $n_j = \sqrt{\varepsilon_j}$ forming the system under study begin ε_j the electric permittivity, which is shown in Fig. 1. In Eq. (1) ω is the frequency of the electromagnetic wave, c is the speed of light in vacuum, and $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ is independent of z.

The function Ψ^j represents the electric or magnetic field and the polarization TE is considered in this work. To solve Eq. (1), we introduce a Green function $G(\mathbf{r},\mathbf{r'})$, as the solution of the equation given by:

$$\nabla^2 G_j(\mathbf{r},\mathbf{r'}) + n_j^2(\omega)\frac{\omega^2}{c^2}G_j(\mathbf{r},\mathbf{r'}) = -4\pi\delta(\mathbf{r}-\mathbf{r'}), \tag{2}$$

where $\delta(\mathbf{r}-\mathbf{r'})$ is the Dirac delta. A Green function that is a solution of Eq. (2) is given by:

$$G_j(\mathbf{r},\mathbf{r'}) = i\pi H_0^1 n_j \left(\frac{\omega|\mathbf{r}-\mathbf{r'}|}{c}\right), \tag{3}$$

With $H_0^1(z)$ the Hankel function of the first kind and zero order. Applying Green's second integral theorem [3, 5] for the functions Ψ and G in each region corresponding to the j-th medium:

$$\Psi_j(\mathbf{r})\theta_j(\mathbf{r}) = \frac{1}{4\pi}\int_{\Gamma_j}\left[G_j(\mathbf{r},\mathbf{r'})\frac{\partial\Psi_j(\mathbf{r'})}{\partial n'} - \Psi_j(\mathbf{r'})\frac{G_j(\mathbf{r},\mathbf{r'})}{\partial n'}\right]ds', \tag{4}$$

where $\theta_j(r)$ is a step function whose values is one for all points in the medium j-th and zero otherwise. In Eq. (4) the surface is bounded by the corresponding closed boundary Γ_j and the normal derivative $\partial/\partial n'$ goes outside the boundary Γ_j. To solve Eq. (4) it is necessary to convert the integro-differential equations into matrix equations by means of a rectangle approximation to evaluate the integrals in small intervals.

Under this consideration, Eq. (4) is transformed into the system of linear equations where matrix elements L^j_{mn} and N^j_{mn} [3]. The property of periodicity that the system has in the x-direction direction is a condition of symmetry that is especially considered. Due to this property and the form of Eq. (1), Bloch's theorem establishes a periodicity condition as $\Psi(x-P,y) = \Psi(x,y)\mathrm{e}^{-iKP}$ with K the Bloch vector.

On the other hand, we have that the boundary conditions along the contours Γ_j are given by $\Psi_n^{(j)} = \Psi_n^{(j+1)} = 0$ and $\partial\Psi_n^{(j)}/\partial n = \partial\Psi_n^{(j+1)}/\partial n$, for TE polarization, where $j = 1$ y 2. With these considerations we find a matrix equations $M(\omega)\ F(\omega) = 0$, which has a representative matrix M and F the source vector, and both depend on the frequency ω and the Bloch vector K. To determine the frequency ω we define the determinant function:

$$D(K,\omega) = \ln|\det(K,\omega)|. \tag{5}$$

Which numerically presents local minimum points that will give us the numerical dispersion relatio, $\omega = \omega(K)$.

3 Photonic Band Structures

In this work we are going to introduce dimensionless values, so our results are expressed in terms of the reduced Bloch vector given by $K_r = (P/2\pi)K$ and the reduced frequency $\omega_r = (P/2\pi)\omega$. The photonic band structures of a PCW with an array of perfectly conducting inclusions involving Koch snowflake fractal structures (see Fig. 1) are shown below.

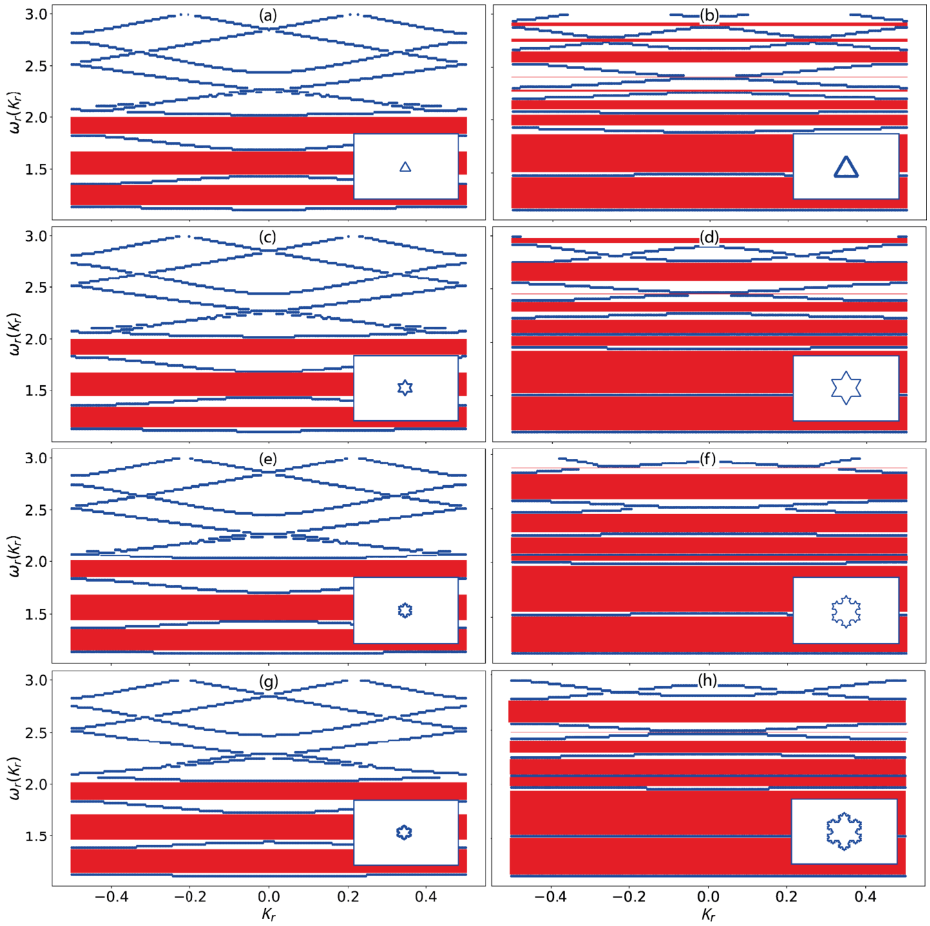

Fig. 2. Photonic band structures of the perfectly conducting PCW that is formed with an array of Koch fractal inclusions of (a) and (b) 0, (c) and (d) 1, (e) and (f) 2, (g) and (h) 3 iterations for the side lengths $L = 1/3$ (first column) and $L = 1$ (second column) of the original triangle. The band gaps are represented by the red stripes. The insets on the right show unit cells in real space whose cross sections of inclusions are made up of Koch snowflake fractals of various orders.

The geometrical values of the waveguide taken into account were: $b = \pi\,\mu$ m and $P = 2\pi\,\mu$ m. Figure 2 shows the band structures of the rectangular lattice with Koch snowflake fractals of $n = 0$, 1, 2 and 3 iterations for the side lengths $L = 1/3$ [Figs. 2(a), (c), (e) and (g)] and $L = 1$ [Figs. 2 (b), (d), (f) and (h)] of the original triangle, respectively.

The results show the appearance of several band gaps (red stripes) as the size of the inclusions and the order of fractal iterations increases, which substantially modify the photonic band structure. Furthermore, it is possible to obtain discrete modes [Figs. 2(d), (f) and (h)] for a given frequency range and then the PCW with Koch fractal structures can act as a unimodal filter.

4 Conclusions

We applied an integral numerical method to calculate the photonic band structures of a PCW formed by two perfectly conducting parallel plates and an array of of inclusions involving Koch snowflake fractal structures. The numerical results obtained show good accuracy and efficiency of the numerical method applied. In addition, it was found that varying the inclusion size for several iterations of the Koch snowflake fractal allows to control the band structure of the system to some extent.

The results show the appearance of several band gaps that substantially modify the photonic band structure. Moreover, it is possible to obtain discrete modes for a certain range of frequencies and then the PCW acts as an unimodal filter. This system is considered as a photonic crystal whose band structures correspond in many respects as a conventional photonic crystal, but using only one material. Therefore, the results of the optical response of a periodic PCW with Koch fractal structures promise excellent and interesting optical applications such as filtering and coding of optical signals.

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